Automorphism groups of rigid affine surfaces: the identity component

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Let Y be an affine variety over an algebraically closed field $\mathbb{K}$ of characteristic zero. We say that Y is *rigid* if it admits no effective action of the additive group of the field $\mathbb{G}_a$.

Conjecture (P.–Zaidenberg'13). *If Y is a rigid affine algebraic variety over $\mathbb{K}$, then the identity component of the automorphism group $\mathrm{Aut}^\circ(Y)$ is an algebraic torus of rank $\leq \dim Y$.*

We prove this conjecture in the case of a normal affine surface Y. We work with minimal completions (X, D) of Y with a normal crossing boundary divisor D and use the theory of birational transformations of the dual graph of the boundary divisor.

A *extremal linear segment* S of the dual weighted graph $\Gamma(D)$ is a maximal linear subgraph, which is either connected to $\Gamma(D) \setminus S$ by an edge from its end vertex or not connected at all. It is called *non-admissible*, if it contains a vertex of non-negative weight, that is, the corresponding curve in D has a non-negative self-intersection index. We present the following theorem.

Theorem (P.–Zaidenberg [1]). *If the surface Y is rigid, then all completions (X, D) have no non-admissible extremal linear segments in $\Gamma(D)$, and the identity component $\mathrm{Aut}^\circ(Y)$ is an algebraic torus of dimension ≤ 2.*

Otherwise, any completion (X, D) has a non-admissible extremal linear segment in $\Gamma(D)$, and $\mathrm{Aut}^\circ(Y)$ contains an infinite-dimensional abelian unipotent subgroup.

We also present a complete list of birational classes of dual graphs, which have a unique minimal model. All other dual graphs admit an infinite number of minimal models. The talk is based on the joint work with Mikhail Zaidenberg [1].

affine surface, automorphism group, completion, normal crossing divisor

References

[1] A. Perepechko, M. Zaidenberg. *Automorphism groups of rigid affine surfaces: the identity component*, preprint, arXiv:2208.09738.

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