Double cosets NgN of normalizers of maximal tori of simple algebraic groups and orbits of partial actions of Cremona subgroups

Nikolai Gordeev

Abstract. Let G be a simple algebraic group over an algebraically closed field K and let $N = N_G(T)$ be the normalizer of a fixed maximal torus $T \leq G$. Further, let U be the unipotent radical of a fixed Borel subgroup B that contains T and let U^- be the unipotent radical of the opposite Borel subgroup B^-. The Bruhat decomposition implies the decomposition $G = NU^-UN$. The Zariski closed subset $U^-U \subset G$ is isomorphic to the affine space A^m_K where $m = \dim G - \dim T$ is the number of roots in the corresponding root system. Here we construct a subgroup $\mathcal{N} \leq \mathrm{Cr}_m(K)$ that "acts partially" on $A^m_K \approx \mathcal{U}$ and we show that there is one-to-one correspondence between the orbits of such a partial action and the set of double cosets $\{NgN\}$. Here we also calculate the set $\{g_\alpha\}_{\alpha\in\mathfrak{A}} \subset \mathcal{U}$ in the simplest case $G = \mathrm{SL}_2(\mathbb{C})$.

References

[1] N. Gordeev, E. Egorchenkova, *Double cosets NgN of normalizers of maximal tori of simple algebraic groups and orbits of partial actions of Cremona subgroups*, arXiv:2112.06332 [math.AG] (https://arxiv.org/abs/2112.06332v1)

[2] F. Abadie, *Partial actions and grouppoids*. Proceedings of the American Mathematical Societey, V.132, N. 4, 1037–1047.

[3] E. Ellers, N. Gordeev, *Intersection of Conjugacy Classes of Chevalley Groups with Gauss Cells*. Journal of Algebra, 220(1999), 591-611.

[4] R.Exel, *Partial actions of groups and actions of inverse semigroups*. Proceedings of the American Mathematical Societey, Volume 126 (1998), N. 12, 3481–3494.

[5] Jean-Pierre Serre, *Le groupe de Cremona e ses sous-groupes finis*, Seminaire BOURBAKI 61, 2008-2009, no. 1000, 1-23.

[6] Vinberg, E. B., Popov, V. L. Invariant theory. (Russian) Algebraic geometry, 4 (Russian), 137–315, Itogi Nauki i Tekhniki, Sovrem. Probl. Mat. Fund. Naprav., 55, Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1989.

Nikolai Gordeev
Department of Mathematics, Russian State Pedagogical University, Moijka 48, St.Petersburg,
191-186, Russia
e-mail: nickgordeev@mail.ru