

Tits-type alternative for automorphism groups

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In 1972, Jacques Tits [3] has proved that any subgroup of the general linear group $GL(n)$ over a field of characteristic zero either contains a nonabelian free subgroup or is a finite extension of a solvable group. In the first part of the talk we survey known facts on the Tits alternative for groups of regular and birational automorphisms of algebraic varieties. Attempts to prove that the Tits alternative holds for automorphism groups have served as an important incentive to study the structure of subgroups. In the last decades this direction became an active research area with many deep results and intriguing open problems.

The second part of the talk is based on joint works with Mikhail Zaidenberg. Let X be an affine algebraic variety defined over an algebraically closed field of characteristic zero and \mathbb{G}_a be the additive group of the ground field. Consider a subgroup H of the automorphism group $\text{Aut}(X)$ generated by a finite collection of \mathbb{G}_a -subgroups U_1, \dots, U_k . We conjecture that either H contains a nonabelian free subgroup or H is a unipotent affine algebraic group. This conjecture is proved in [1] under assumptions that X is an affine toric variety and the subgroups U_1, \dots, U_k are normalized by the acting torus. In [2], we show that if X is a complex affine surface then either H contains a nonabelian free subgroup or H is a metabelian unipotent affine algebraic group.

References

- [1] Ivan Arzhantsev and Mikhail Zaidenberg. Tits-type alternative for groups acting on toric affine varieties. *Int. Math. Res. Not. IMRN* 2022 (2022), no. 11, 8162-8195
- [2] Ivan Arzhantsev and Mikhail Zaidenberg. Tits-type alternative for certain groups acting on algebraic surfaces. <https://arxiv.org/abs/2111.06659>
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